Beam Single Spin Asymmetries in Electron-Proton Scattering

Pratik Sachdeva[1], Wally Melnitchouk[2], Peter Blunden[3]

Abstract

The Q-weak experiment at Jefferson Lab aims to provide a precision test of the Standard Model by measuring the weak charge of the proton, Q_W^P, through electron-proton scattering. In Q-weak, a longitudinally polarized electron beam was scattered off an unpolarized proton target. The difference between the cross sections of the spin up and spin down cases results in a beam single spin asymmetry (BSSA), which can be used to find Q_W^P. These BSSA may be longitudinal, transverse, or normal, depending on the polarization of the electrons. After Q-weak's commissioning run, experimentalists were concerned that unconsidered BSSA effects may have introduced a displacement in their measurements of the BSSA. To determine if there was a displacement, we computed the beam transverse single spin asymmetry (BTSSA) that results from the interference of the one-photon and Z exchange amplitude and compared this to the beam normal single spin asymmetry (BNSSA), which results from the interference of the one- and two-photon exchange amplitudes. We computed these quantities in the near forward limit and used deep inelastic structure functions to evaluate the BNSSA. It was found that the combination of the BNSSA and the BTSSA results in a small phase shift in the total asymmetry. Furthermore, the inelastic hadronic intermediate state provides a sizable contribution to the BNSSA. The phase shift, however, is too small to be detectable, implying that the Q-weak measurements for the BNSSA are accurate. Despite the small magnitude of the phase shift, however, it may be detectable in the future experiments at higher energies, such as the Electron-Ion Collider (EIC). Furthermore, the framework we developed could potentially be used to evaluate BSSA in other experimental settings in the near forward limit.

[1] *Washington University in St. Louis*
[2] *Jefferson Laboratory, Theory Center*
[3] *University of Manitoba*

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1. Introduction

Polarized electron beams often provide important opportunities to probe the structure of the nucleon. Scattering experiments at Jefferson Laboratory often involve such polarized electron beams. The polarization of the electron beam can affect the resulting cross section of the experiment. This has the potential to introduce asymmetries which result from the differences in cross section produced by the spin up and spin down polarizations. These asymmetries are observables which can provide valuable information about the system. For example, the Q-weak experiment, performed at Jefferson Lab, aims to perform a precision test of the Standard Model by measuring the weak charge of the proton. This is done through a parity-violating asymmetry.

The Q-weak experiment, and others similar to it, use a polarized electron beam and an unpolarized electron target. Thus, the asymmetries produced in this experiment are known as beam single spin asymmetries (BSSA). These BSSA vanish for one-photon exchange processes. We must examine higher order processes such as two-photon exchange or parity-violating effects in order to obtain these asymmetries.

In this project, we investigate BSSA closely in the Q-weak regime. This regime is characterized by low momentum transfer and very forward scattering angles. We perform our calculations in the *near-forward limit* for very small Q^2. The framework we develop could potentially be used for other scattering experiments performed at similar kinematics.

2. Beam Single Spin Asymmetries

2.1 Definition

In electron-proton scattering experiments, the resulting cross section depends on the spin of the involved particles. In the case of Q-weak, a polarized beam scatters off an unpolarized target. The difference in cross sections between the parallel and anti-parallel polarizations introduces an observable called a **beam single spin asymmetry (BSSA)**. These asymmetries can be used to determine useful quantities in the system. For example, in Q-weak, an asymmetry A_{PV} resulting from parity-violating effects can be used to find the weak charge of the proton, Q_W^P.

The beam single spin asymmetry (BSSA) B is defined as

$$B = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \tag{1}$$

where $\sigma^{\uparrow}$ ($\sigma^{\downarrow}$) is the cross section of the spin up (down) orientation. The BSSA may arise from a variety of exchange effects. In elastic scattering, the BSSA disappears for one-photon exchange, so we must look at higher order effects to find these asymmetries. We can write the scattering amplitude as

$$\begin{aligned} |\mathscr{M}|^2 &= |\mathscr{M}_\gamma + \mathscr{M}_Z + \mathscr{M}_{\gamma\gamma} + \cdots|^2 \\ &= |\mathscr{M}_\gamma|^2 + \mathscr{M}_\gamma^* \mathscr{M}_Z + \mathscr{M}_\gamma^* \mathscr{M}_{\gamma\gamma} + \cdots \end{aligned}$$

Since B disappears for $|\mathscr{M}_\gamma|^2$, the leading order terms to consider are $\mathscr{M}_\gamma^* \mathscr{M}_{\gamma\gamma}$ and $\mathscr{M}_\gamma^* \mathscr{M}_Z$. Both these terms contribute to the beam single spin asymmetry. This contribution varies depending on the polarization of the electron.

2.2 Transverse and Normal Polarizations

The polarization of the electron beam will determine what exchange amplitudes introduce BSSA in the scattering process. Electron beams can be polarized *longitudinally*, where the spin lies along the beam direction, *transversely*, where the spin lies perpendicular to the beam direction, or *normally*, where the spin lies perpendicular to the scattering plane.

The Q-weak experiment's main objective was to measure a parity-violating asymmetry A_{PV} which results from the longitudinal polarization of the electron beam. The Q-weak experiment also measured a **beam normal single spin asymmetry (BNSSA)**: a BSSA resulting from the normal component of a transversely polarized beam.

For this project, we are concerned with a transversely polarized beam, as shown below.

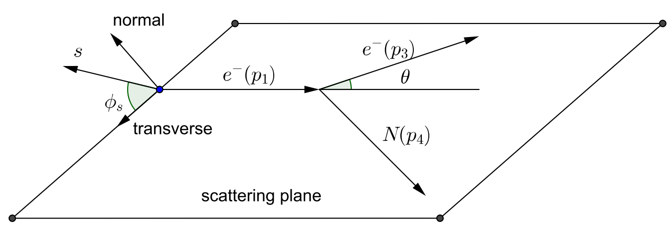

Figure 1. Electron Scattering Plane

In the above figure, the transverse spin makes an angle ϕ_s with the scattering plane. This allows us to split the spin into its normal and transverse components, which lie perpendicular to the scattering plane and beam direction, respectively. Each of these components introduce their own asymmetries: the normal component introduces the aforementioned BNSSA, which behaves as $B_n \sim \sin\phi_s$, and the transverse component introduces a **beam transverse single spin asymmetry (BTSSA)**, which behaves as $B_t \sim \cos\phi_s$.

2.3 Motivation

As discussed in the previous section, the BNSSA measured in Q-weak has a sinusoidal dependence for a general transverse spin. This sinusoidal dependence is shown in Figure 2 [1].

In calculating this BNSSA, experimentalists were concerned that unconsidered BSSA effects, such as a

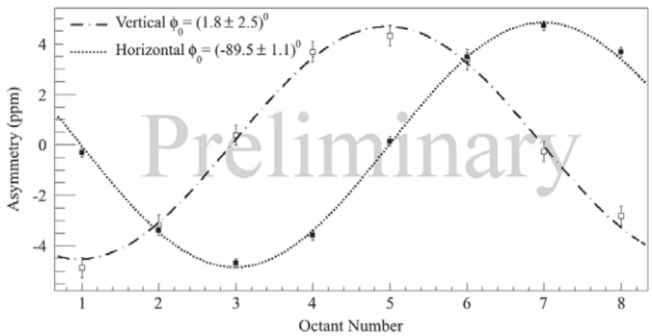

Figure 2. BNSSA for Q-weak (Preliminary)

BTSSA, could add a phase shift to the results. To determine if this was the case, it became necessary to better understand BSSA and the exchange effects that produce them.

To approach this problem,

- We computed the beam transverse single spin asymmetry (BTSSA) that results from the interference of the one-photon and Z exchange amplitude.
- We compared this BTSSA to the beam normal single spin asymmetry (BNSSA), which results from the interference of the one- and two-photon exchange amplitudes.
- We also computed BNSSA in the case of an inelastic hadronic intermediate state as a comparison to the Q-weak measurement.

We computed these quantities in the near forward limit and used deep inelastic structure functions to evaluate the BNSSA.

3. BSSA in Elastic Scattering

3.1 Formalism

We first review the formalism for the beam single spin asymmetry set up by [2, 3]. Consider the kinematic process for elastic lepton-nucleon scattering,

$$e^-(p_1)+N(p_2)\to e^-(p_3)+N(p_4), \tag{2}$$

with electron mass m_e and hadron mass M. We use the kinematic variables

$$P=\frac{p_2+p_4}{2}, \qquad K=\frac{p_1+p_3}{2}, \qquad q=p_1-p_3,$$

the momentum transfer $Q^2=-q^2$, and the dimensionless invariants

$$\tau=\frac{Q^2}{4M^2}, \qquad \nu=\frac{P\cdot K}{M^2}, \qquad \varepsilon=\frac{\nu^2-\tau(1+\tau)}{\nu^2+\tau(1+\tau)}.$$

The general amplitude for the elastic scattering process (1) can be divided into six invariant amplitudes [4]. Three of these amplitudes flip the helicity of the electron while the other three do not. Thus, the total amplitude can be written as

$$T=T_{\text{flip}}+T_{\text{non-flip}}.$$

The flip (non-flip) amplitudes T_{flip} ($T_{\text{non-flip}}$) are linear combinations of these invariants and can be written as [2]

$$\begin{aligned}T^{\text{non-flip}} =& \frac{e^2}{Q^2}\bar{u}_e(p_3)\gamma_\mu u_e(p_1)\cdot\bar{u}_N(p_4)\\ &\times\left(\widetilde{G}_M\gamma^\mu-\widetilde{F}_2\frac{P^\mu}{M}+\widetilde{F}_3\frac{\not{K}P^\mu}{M^2}\right)u_N(p_2)\end{aligned} \tag{3}$$

and

$$\begin{aligned}T_{\text{flip}} = \frac{e^2}{Q^2}\frac{m_e}{M}\Bigg[&\bar{u}_e(p_3)u_e(p_1)\cdot\bar{u}_N(p_4)\left(\widetilde{F}_4+\widetilde{F}_5\frac{\not{K}}{M}\right)u_N(p_2)\\ &+\widetilde{F}_6\bar{u}_e(p_3)\gamma_5 u_e(p_1)\cdot\bar{u}_N(p_4)\gamma_5 u_N(p_2)\Bigg].\end{aligned} \tag{4}$$

Here, the generalized form factors $\widetilde{G}_M, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5$, and $\widetilde{F}_6$ are complex functions of ν and Q^2. In the Born approximation, they reduce to

$$\begin{aligned}\widetilde{G}_M^{\text{Born}}(\nu,Q^2)&=G_M(Q^2)\\ \widetilde{F}_2^{\text{Born}}(\nu,Q^2)&=F_2(Q^2)\\ \widetilde{F}_{3,4,5,6}^{\text{Born}}(\nu,Q^2)&=0.\end{aligned}$$

Lastly, we can define one more generalized form factor $\widetilde{G}_E$ as $\widetilde{G}_E=\widetilde{G}_M-(1+\tau)\widetilde{F}_2$. In the Born approximation, it reduces to $\widetilde{G}_E^{\text{Born}}=G_E(Q^2)$.

3.2 Beam Normal Single Spin Asymmetry

For a spin polarization S_n^μ normal to the scattering plane, i.e.,

$$S_n^\mu=(0,\vec{S}_n), \qquad \vec{S}_n=\frac{\vec{p}_1\times\vec{p}_3}{|\vec{p}_1\times\vec{p}_3|},$$

a beam normal single spin asymmetry B_n results from the interference between one-photon exchange and the general scattering amplitude. The leading order contribution comes from the interference between the one- and two-photon exchange amplitudes and is of order e^2. B_n is given as [2]

$$B_n = \frac{2m_e}{Q}\sqrt{2\varepsilon(1-\varepsilon)}\sqrt{1+\frac{1}{\tau}}\left(G_M^2+\frac{\varepsilon}{\tau}G_E^2\right)^{-1} \times\left\{-\tau G_M \operatorname{Im}\left(\widetilde{F}_3+\frac{1}{1+\tau}\frac{\nu}{M^2}\widetilde{F}_5\right) -G_E \operatorname{Im}\left(\widetilde{F}_4+\frac{1}{1+\tau}\frac{\nu}{M^2}\widetilde{F}_5\right)\right\}, \quad (5)$$

This BNSSA behaves linearly as the electron mass and vanishes in the Born approximation.

We calculated the BNSSA in the case of a general transverse spin

$$\vec{S} = \cos\phi_s\,\hat{x}+\sin\phi_s\,\hat{y}$$

where ϕ_s is the azimuthal angle with respect to the scattering plane. We found that the interference between the one-photon exchange and general amplitude produces a BSSA dependent only on the normal component of the spin. This asymmetry $B_{n,\text{ gen}}$ is equal to

$$B_{n,\text{ gen}} = B_n \sin\phi_s,$$

where B_n is defined in (4). Here we find the sinusoidal dependence that is reflected in the Q-weak measurements.

3.3 Beam Transverse Single Spin Asymmetry

Using the formalism in the previous section, we found that a BTSSA results from the transverse component of the spin due to the interference of the one-photon and Z exchange amplitudes. For a strictly transverse spin $\vec{S}=\hat{x}$, this transverse asymmetry is

$$B_t = \frac{8G_F m_e M^3}{e^2(s-M^2)}\sqrt{\frac{\varepsilon(1-\varepsilon)}{\tau+1}}\left(G_M^2+\frac{\varepsilon}{\tau}G_E^2\right)^{-1} \times\left\{g_A^e\left(G_M G_A^Z\tau(\tau-\nu+1)-G_E G_E^Z(\nu+\tau-1)\right) - g_V^e G_M G_M^Z \tau(\tau+1)\right\}, \quad (6)$$

where g_A^e and g_V^e are the axial and vector couplings respectively, G_A^Z, G_M^Z and G_E^Z are the weak form factors, G_F is Fermi's coupling constant, and $s=(p_1+p_2)^2$.

If we calculate the BTSSA in the case of a general spin, we pick up the sinusoidal dependence through the transverse component. Thus, we can write the general transverse asymmetry as

$$B_{t,\text{ gen}} = B_t\cos\phi_s,$$

similar to the BNSSA.

3.4 Combination of Asymmetries

Both the BNSSA and BTSSA calculations have sinusoidal dependence. Thus, if we combine both effects, we have a total asymmetry B which should have some periodic structure. Taking the general BNSSA and BTSSA in combination gives

$$\begin{aligned} B &= B_{n\text{ gen}}+B_{t\text{ gen}} \\ &= B_n\sin\phi_s + B_t\cos\phi_s \\ &= \sqrt{B_n^2+B_t^2}\,\sin(\phi_s+\delta). \end{aligned} \quad (7)$$

Here,

$$\delta = \tan^{-1}\left(\frac{B_t}{B_n}\right). \quad (8)$$

Thus, a measurement of the BNSSA as in Q-weak will retain the sinusoidal dependence but contain a phase shift, which the experimentalists were concerned about. We now turn to the magnitude of this phase shift and consider whether it is detectable.

The preliminary result for B_n as measured by Q-weak is [1]

$$B_n = -5.350\pm 0.067_{\text{stat}}\pm 0.137_{\text{sys}}\text{ ppm}.$$

Q-weak kinematics give us $Q^2=0.025$ GeV^2 and electron beam energy $E_1 = 1.155$ GeV. Using these values, we can calculate an approximate value of B_t as

$$B_t \approx 1.116\times 10^{-5}\text{ ppm}.$$

This gives us

$$\begin{aligned} |\delta| &= \left|\tan^{-1}\left(\frac{B_t}{B_n}\right)\right| \approx \left|\frac{B_t}{B_n}\right| \\ &= 2.086\times 10^{-6}. \end{aligned}$$

At this magnitude, the phase shift is too small to affect Q-weak measurements.

3.5 Order of Magnitude Estimates

We now give order of magnitude estimates of the various asymmetries discussed and consider the leading effects to these asymmetries.

1. **Longitudinal Asymmetry** (A_{PV}): The major contribution comes from the ratio of Z-boson to photon propagators in the interference of the Born photon and Z-boson exchange amplitudes, which gives
$$A_{PV}\sim\frac{Q^2}{M_Z^2}\approx Q^2\times 10^{-4}.$$

2. **BNSSA** (B_n): The normal asymmetry is of order e^2, and thus has a factor of α. Furthermore, transversely polarizing the beam adds a factor of $\frac{m_e}{M}$. We have

$$B_n \sim \frac{\alpha m_e}{M} \approx 5\times 10^{-6}.$$

3. **BTSSA** (B_t): Here we have both the contribution from parity-violating effects and the beam polarization.

$$B_t \sim Q^2 \frac{m_e}{M_Z^2 M} \approx Q^2 \times \left(5\times 10^{-8}\right).$$

Thus, the small magnitude of B_t comes from the combination of the beam polarization and parity-violating effects. This transverse asymmetry could potentially be amplified at higher momentum transfers, indicated by the Q^2 dependence of B_t.

4. Inelastic Hadronic Intermediate State Contribution to BNSSA

4.1 Two-Photon Exchange Contribution

Pasquini and Vanderhaeghen [5] showed that the inelastic hadronic intermediate state has a large contribution to the BNSSA for two-photon exchange processes. Thus, it is useful to consider this contribution in the near-forward limit as is the case for Q-weak. We consider the same elastic scattering process (2). As shown in Figure 3, the intermediate electron, taken on-shell, has momentum k while the exchanged photons have momenta q_1 and q_2. We further define the momentum transfers

$$Q_1^2 = -q_1^2, \qquad Q_2^2 = -q_2^2,$$

and the intermediate hadronic mass

$$W^2 = (p_2+q_1)^2.$$

The BNSSA B_n, as stated before, has a leading order contribution from the interference of the one- and two-photon exchange amplitudes. In the previous section, we used the general scattering amplitude to compute B_n. Here we calculate it explicitly through the two-photon exchange amplitude.

The contribution to B_n comes from the absorptive part of $\mathscr{M}_{\gamma\gamma}$. We can write B_n as [3]

$$B_n = \frac{2\,\mathrm{Im}\left(\sum_{\text{spins}} \mathscr{M}_\gamma^* \cdot \mathrm{Abs}\,\mathscr{M}_{\gamma\gamma}\right)}{\sum_{\text{spins}} |\mathscr{M}_\gamma|^2}. \tag{9}$$

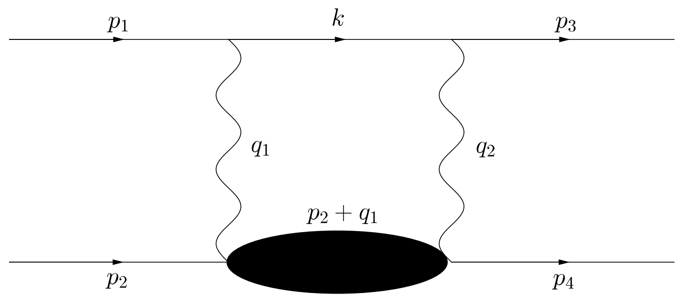

Figure 3. Two Photon Exchange Diagram

Here, the absorptive part of $\mathscr{M}_{\gamma\gamma}$ can be written as [5]

$$\mathrm{Abs}\,\mathscr{M}_{\gamma\gamma} = \frac{e^4}{(2\pi)^3}\int \frac{d^3\vec{k}}{2E_k}\,\frac{1}{Q_1^2 Q_2^2}\,L_{\mu\nu}W^{\mu\nu}, \tag{10}$$

where E_k is the energy of the intermediate electron, $L_{\mu\nu}$ is the lepton tensor, and $W^{\mu\nu}$ is the hadron tensor.

4.2 Lepton-Hadron Tensor Contraction

The interference between the one- and two-photon exchange amplitudes produces 3-rank leptonic and hadronic tensors. The leptonic tensor is straightforward and can be written as

$$L_{\alpha\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(1+\gamma_5 \not{s})(\not{p}_1+m_e)\gamma_\alpha(\not{p}_3+m_e)\gamma_\mu(\not{k}+m_e)\gamma_\nu\right]. \tag{11}$$

The hadronic tensor is

$$H^{\alpha\mu\nu} = W^{\mu\nu}\cdot\left[\overline{u}(p_4)\Gamma^\alpha(p_2,p_4)u(p_2)\right]^*. \tag{12}$$

In order use a near-forward limit approximation, we take the forward limit on the *hadronic* tensor. This allows us to write $H^{\alpha\mu\nu}$ as

$$H^{\alpha\mu\nu} = 2p_2^\alpha\, W^{\mu\nu}_{(\text{inelas})},$$

where $W^{\mu\nu}_{(\text{inelas})}$ is given as [3]

$$W^{\mu\nu}_{(\text{inelas})} = 2p_2^\alpha\left\{\left(\frac{q_1^\mu q_1^\nu}{q_1^2} - g^{\mu\nu}\right)W_1 + \frac{1}{M^2}\left(p_2^\mu - \frac{p_2\cdot q_1}{q_1^2}q_1^\mu\right)\left(p_2^\nu - \frac{p_2\cdot q_1}{q_1^2}q_1^\nu\right)W_2\right\} \tag{13}$$

The asymmetry requires the spin-dependent part of the lepton-hadron contraction. Thus, we must take the imaginary part. This gives us

$$\mathrm{Im}L_{\alpha\mu\nu}H^{\alpha\mu\nu} = \frac{8m_e}{M^2Q_1^2}\Big[2W_1M^2Q_1^2\left(\varepsilon^{p_1p_2qs}+\varepsilon^{p_2qq_1s}\right) - W_2\varepsilon^{p_2qq_1s}\left((W^2-M^2-Q_1^2)(p_1\cdot p_2)+M^2Q_1^2\right)\Big]. \tag{14}$$

where ε is the Levi-Civita tensor. This tensor contraction is frame independent.

4.3 Evaluating the BNSSA

Using equation (9) and (10), we can write B_n as

$$B_n = \frac{e^6}{(2\pi)^3 Q^2}\left(\sum_{\text{spins}} |\mathscr{M}_\gamma|^2\right)^{-1} \times \int \frac{d^3\vec{k}}{E_k}\frac{1}{Q_1^2 Q_2^2}\,\text{Im}\left\{L_{\alpha\mu\nu}H^{\alpha\mu\nu}\right\}. \tag{15}$$

Letting

$$D(s,Q^2) = \frac{8Q^4}{1-\varepsilon}\left\{G_M^2 + \frac{\varepsilon}{\tau}G_E^2\right\},$$

we can write the BNSSA as

$$B_n = \frac{1}{(2\pi)^3}\frac{e^2 Q^2}{D(s,Q^2)}\int \frac{d^3\vec{k}}{E_k}\frac{1}{Q_1^2 Q_2^2}\,\text{Im}\left\{L_{\alpha\mu\nu}H^{\alpha\mu\nu}\right\}.$$

Let θ_k and ϕ_k be the polar and azimuthal angles of the intermediate electron, respectively. Then, we can rewrite the phase space integral in spherical coordinates. Using the relations:

$$Q_1^2 = 2E_k E_1(1-\cos\theta_k) \tag{16}$$

$$Q_2^2 = 2E_k E_3(1-\cos\theta\cos\theta_k - \sin\theta\cos\phi_k\sin\theta_k), \tag{17}$$

where E_k is the energy of the intermediate electron and θ is the lab scattering angle, we can write B_n as

$$B_n = -\frac{1}{(2\pi)^3}\frac{e^2Q^2}{D(s,Q^2)}\frac{1}{8E_1E_3M}\int_0^{2\pi} d\phi_k \int_{M^2}^{s} dW^2 \times \int_0^{Q^2_{1,\max}} \frac{\text{Im}\left\{L_{\alpha\mu\nu}H^{\alpha\mu\nu}\right\}\, dQ_1^2}{Q_1^2 E_k\left(1-\cos\theta\cos\theta_k - \sin\theta\sin\theta_k\cos\phi_k\right)} \tag{18}$$

Numerical results for B_n in the Q-weak regime will be presented in [6].

5. Conclusion

In this project, we explored beam single spin asymmetries, focusing on the Q-weak regime to determine if there was a displacement in the calculation of the BNSSA. We found that a BNSSA is produced by $\mathscr{M}_\gamma^*\mathscr{M}_{\gamma\gamma}$ while a BTSSA is produced by $\mathscr{M}_\gamma^*\mathscr{M}_Z$.

The combination of these two asymmetries results in a total asymmetry that behaves sinusoidally with a phase shift. This phase shift, however, is too small to be detectable, implying that the Q-weak measurements are accurate. This phase shift may be detectable in the future, however, at experiments involving higher energies, such as the Electron-Ion Collider (EIC). This is shown by the Q^2-dependence of the B_t calculation.

Furthermore, we calculated the contribution to B_n from the interference of the one- and two-photon exchange amplitudes in the case of the an inelastic hadronic intermediate state. Pasquini and Vanderhaeghen showed that this provides a sizable contribution to the total B_n, and it was important to consider its contribution in the Q-weak regime.

We calculated B_n in the near-forward limit by taking the forward limit on the hadronic tensor. We are currently evaluating B_n to see what impact this approximation has on the result. The framework that we developed could potentially be used to evaluate BSSA in other experimental settings in the near forward limit.

Acknowledgments

I would like to thank Jefferson Laboratory and the Department of Energy for providing the opportunity to work on this project.

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