A Discrepancy between Two Criteria of Stability for Hybrid Stars

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Introduction

Neutron stars are stellar structures that can result from the gravitational collapse of a massive star. They are among the most dense, stable structures in the universe. Their stability can be disrupted, however, by radial oscillations, which result when the star is perturbed from its equilibrium configuration. Here we consider **hybrid stars**, or stars characterized by a quark matter core surrounded by a nuclear matter envelope.

John Bardeen, in his *Catalogue of Methods* [1], detailed two criteria for the stability of a compact star. These methods have been shown to agree consistently for regular neutron stars. We observed, however, that the criteria disagreed for a white dwarf with a strange quark core proposed by Norman Glendenning [2]. This observation provided the motivation for this project: to examine whether this discrepancy also occurs for hybrid stars. Such conclusions could provide insight on the possible existence of these stars.

Hybrid Stars

Families of stars are characterized by an **equation of state** (EoS), a relation between the energy density of the star, ϵ, and the pressure, p. Specific stars in these families are uniquely identified by their **central pressure**, p_c. Hybrid stars either have a kink or discontinuity in the equation of state, which represents the change from quark matter to nuclear matter.

Figure 1(a): Continuous Equation of State

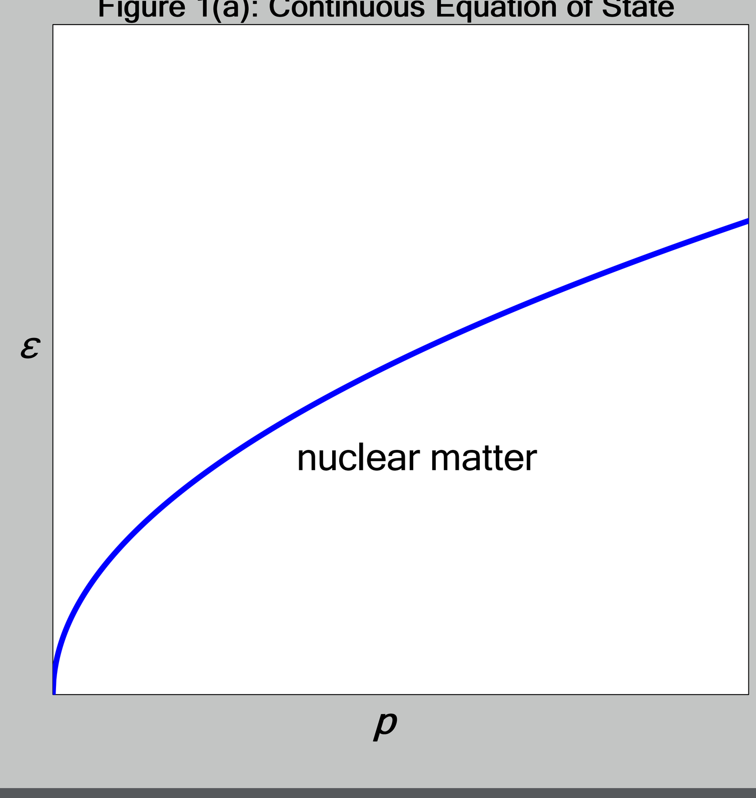
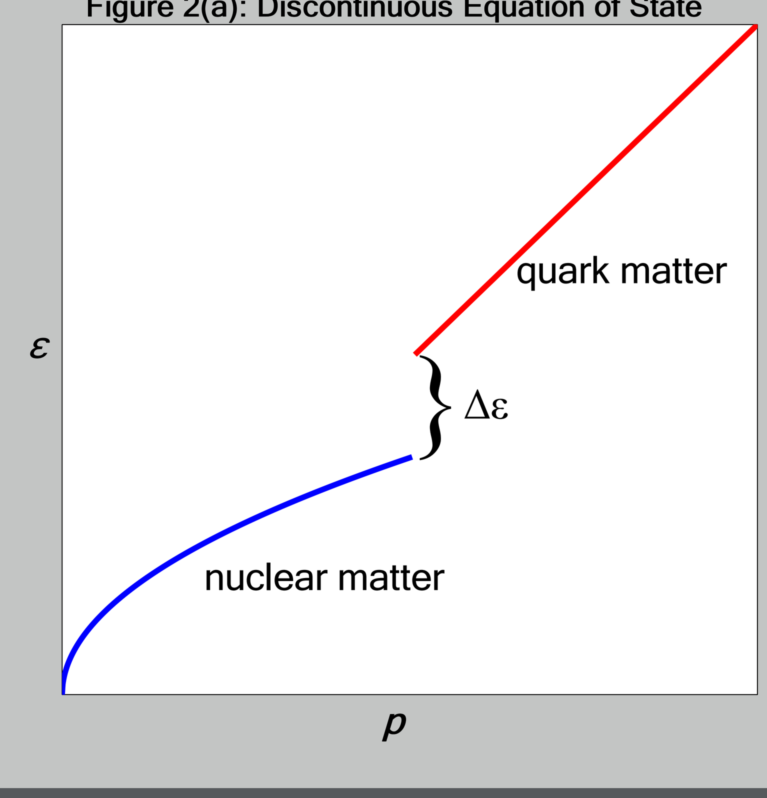


Figure 2(a): Discontinuous Equation of State

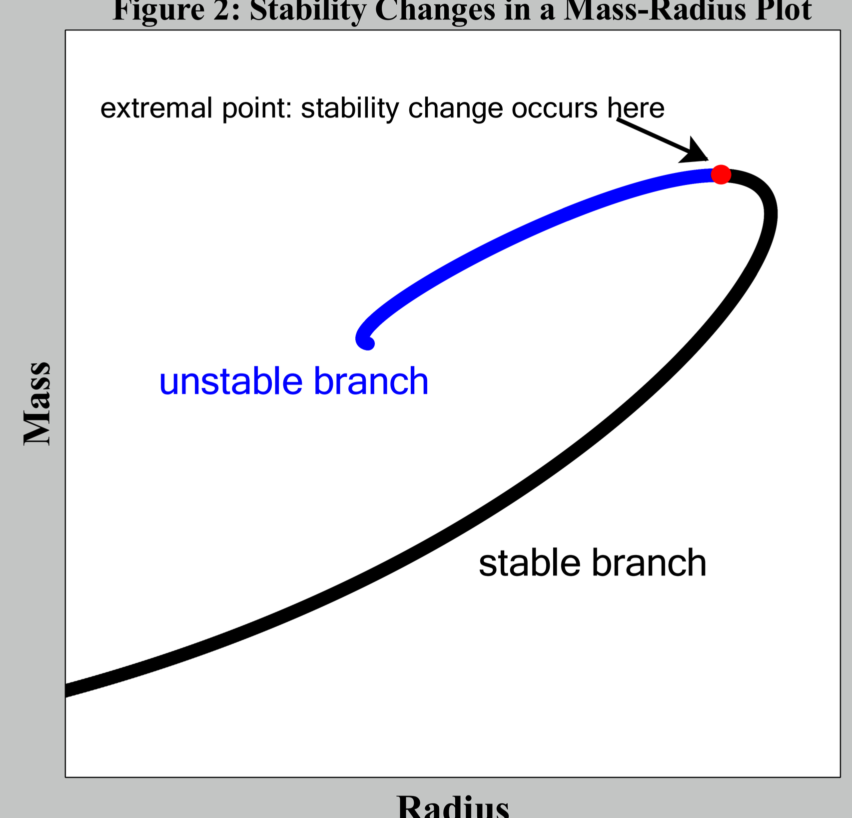


Criteria of Stability

- **(I) Mass-Radius Plots**

This approach is qualitative. For a given EoS, we can vary the central pressure to obtain a sequence of stars. If we plot the mass vs. radius of these stars, we obtain a curve. Below, we show the curve for a linear equation of state.

Figure 2: Stability Changes in a Mass-Radius Plot



Bardeen's criteria states that stability changes occur at **extremal** points in M. Specifically, at a local extremum, when $\frac{dM}{dR} = 0$,

(i) If the MR-plot bends **counterclockwise**, then one previously stable mode becomes **unstable**.

(ii) If the MR-plot bends **clockwise**, one previously unstable mode becomes **stable**.

- **(II) Chandrasekhar's Equation**

We can directly calculate the frequency of oscillations in a neutron star with Chandrasekhar's Equation, a **Sturm-Liouville Problem**. In this problem, the frequency, ω_n^2, must be chosen such that differential equation

$$\frac{d}{dr}\left(\Pi(r)\frac{du_n(r)}{dr}\right) + \left(Q(r) + \omega_n^2 W(r)\right) u_n(r) = 0$$

satisfies the boundary conditions $u_n \sim r^3$ near the origin and $\frac{du_n}{dr} = 0$ at the surface of the star. Here, $u_n(r)$ is the amplitude of the radial oscillations, while Π, Q, and W are all functions dependent on properties of the star.

The solutions to the Sturm-Liouville problems ω_n^2 are called the **radial modes**. If $\omega_n^2 < 0$, the mode is **unstable**. Otherwise, if $\omega_n^2 \geq 0$, the mode is **stable**.

- **Bardeen's theorem states that a stellar structure has $\omega_0^2 = 0$ if and only if $\frac{dM}{dR} = 0$ at this star.**

Method

We used an equation of state of the form

$$\epsilon(p) = \begin{cases} 1.41p + 2 \times 10^8, & p \leq 1.04 \times 10^8 \\ k_{QM}p + \epsilon_{QM}, & p > 1.04 \times 10^8 \end{cases}$$

We have two cases:

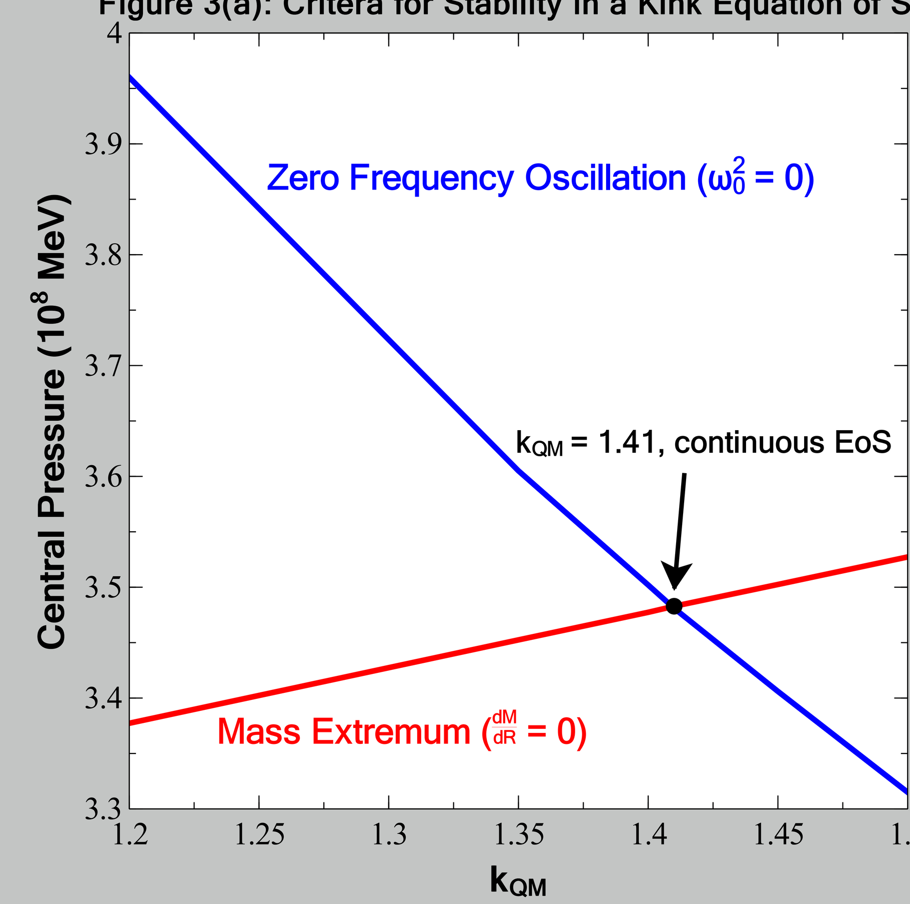
- **(I) Kink:** Here, we vary k_{QM} and choose ϵ_{QM} such that $\epsilon(p)$ is continuous at $p = 1.04 \times 10^8$. Thus, $\epsilon'(p)$ will be discontinuous.
- **(II) Discontinuous:** We set $k_{QM} = k_{NM} = 1.41$ for simplicity, but vary ϵ_{QM} to produce a "jump" in the equation of state.

For each EoS, we calculated the central pressure at which $\omega_0^2 = 0$ and the central pressure at which $\frac{dM}{dR} = 0$ (mass extremum).

Results

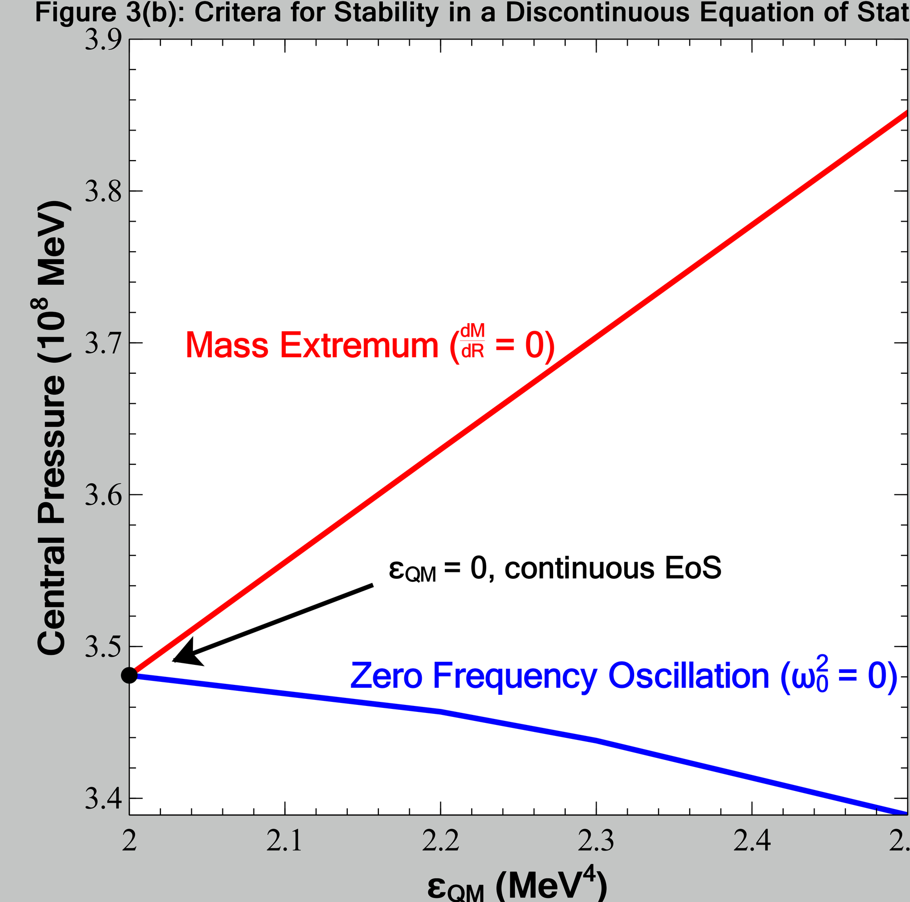
We calculated the central pressures at which $\frac{dM}{dR} = 0$ and $\omega_0^2 = 0$. In both cases of a kink and discontinuity, we found that these central pressures did not agree, violating Bardeen's Theorem. Below, we plot the central pressures as we vary k_{QM} for the kink case:

Figure 3(a): Critera for Stability in a Kink Equation of State



On both sides of $k_{QM} = 1.41$, the central pressures diverge in a consistent manner. Varying ϵ_{QM}, the discontinuous case, gives the plot below. Note that we must have $\epsilon_{QM} > 2 \times 10^8$, as the energy density cannot decrease with respect to pressure.

Figure 3(b): Critera for Stability in a Discontinuous Equation of State



Conclusion

From these results, we can conclude

1. Introducing a kink or discontinuity in the equation of state causes Bardeen's two conditions of stability to disagree as they stand. In order to reconcile this, either:
2. The Chandrasekhar Equation must be modified to produce the correct zero frequency configuration;
3. Or the stability changes for these stars do not occur at mass extremum.

Currently, we are inclined to believe that the Chandrasekhar equation must be modified.

References

[1] J. M. Bardeen K. S. Thorne, D. W. Meltzer.
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Astrophysical Journal, 145:505–513, February 1966.

[2] N.K. Glendenning, Ch. Kettner, and F. Weber.
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